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A Discrete Time (a, c, d) Policy Bulk Service Queue with Accessible and Non-accessible Batches

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ABSTRACT

In this paper we consider a discrete time (a, c, d) policy bulk service queue with accessible and non-accessible batches. Here the server begins service only if the number of units in the queue (n) is at least c and serves a maximum of d units in a batch, with or without accessibility to the batches of ongoing service according as the on going service batch size is less than b or not. The server continue to serve even when the system size is less than c but not less than a secondary limit $a(a \leq c \leq b \leq d)$, after a service completion epoch. The analysis of the model is considered and the steady state probabilities, expected queue length, expected cost function and optimality problem are obtained.

Keywords: *Discrete time queues, (a, c, d)policy, Bulk service queue, Single server, Accessible batches, Steady state probabilities, Expected queue length, Expected cost function.*

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1 Introduction

There are many real life queueing situations in which jobs are served with a control limit policy. For example, in some manufacturing systems it is possible to process jobs manually or by a machine. Up to some control limits, it may be profitable to process the jobs manually one by one, and when the number of jobs exceeds that control limit, processing in a batch by a machine may be cheeper. Batch service models are useful to investigate the performance of various processes in production, manufacturing, traffic signal and telecommunication systems, where the processor processes packets in a batch without over loading the system. Discrete time queueing models are better suited than their continuous time counterparts to evaluate performance measures such as packet loss and packet delay in computer and digital telecommunication networks, because of the clock-driven operation of those systems.

The study of finite buffer queueing models is important rather than infinite buffer queues because in real applications we usually observe limited waiting space. However from a theoretical view point both finite and infinite buffer queueing models have importance. Most of the investigations which employed bulk service models have considered continuous time queues. See Chaudhry and Templeton (1983), Dshalalow(1997) and the references therein for a detailed account of such models. Gupta and Goswami(2002) discussed analytic and computational aspects of $Geo/G^{a,b}/1/N$ queue. Chaudhry and Chang(2003) considered a $Geo/G1^{Y}/1/N$ queue with variable service capacity. They discussed the computational aspects of the distributions of the customers in the queue at various epochs.

Queueing systems with accessible and non accessible batches were considered by many researchers in the past (see for example: Kleinrock(1975), Chiamsiri and leonard(1981), Sivasamy(1990), Gross and Harris(1999), Baburaj and Manoharan(1999), Baburaj(2000), Goswami and Samanta(2009) etc.). Baburaj and Surendranath (2006) considered an (a, c, d) policy $M/M/1$ bulk service queue with accessible and non accessible batches. In the model considered by Baburaj and Surendranath (2005), the server starts service only when the queue size is at least c

and continue to serve even when the system size is less than c but not less than a secondary limit $a(a \leq c)$, after a service completion epoch, such a rule is called Modified General Bulk Service Rule (Modified GBSR). Baburaj and Surendranath (2006) further assumed that the server allows the late entries to join a batch in course of ongoing service till the batch size is less than the accessible limit $b(a \leq c \leq b \leq d)$.

Batch service queueing models have potential applications in many areas, for example; in the loading and unloading of cargos at a seaport, in traffic signal systems, in communication systems (each message consisting of different packets), in transportation systems (buses, guided tours, medical evaluation systems, etc.), in manufacturing systems (electrical, electronic and mechanical industries making products such as cars, computers, machines, etc., where each job is considered as the accumulation of different tasks), in a computer network where jobs are processed in batches. In typical batch service queueing models, once the service is started, arriving unit cannot enter the service station though enough space is available to accommodate it. But in many practical situations the arriving units will be considered for service with current batch in service with some limitation, for example cinema hall, elevator, etc. It may be noted that the accessible batch service has more economic utilizations in providing better service to the queue. For example, in many shuttle transportation systems, we observe units being transported according to accessibility rule with some limitation.

Here we consider an (a, c, d) policy discrete time bulk service queue with accessible and non accessible batches. In this model the customers arrive according to a poisson process with parameter λ. Here the server allows late arrivals to join the ongoing service till the batch size is less than b (accessible batch). In this model the server continue to serve even when the queue size n is less than c but not less than a secondary limit $a(a \leq c \leq b \leq d)$, after a service completion epoch. The service time distribution is assumed to be exponential with parameter μ. Section 2 provides the description of the model. Section 3, presents the analysis of the model with infinite buffer. The transient and steady state behaviour of the model is studied in subsection 3.1 . Explicit expressions for the expected queue length and expected

cost function are given in subsections 3.2 and 3.3 respectively. In subsection 3.4, particular cases of the model are discussed. The model with finite buffer is studied in section 4. Subsections 4.1 and 4.2 respectively gives the steady state probabilities and expected queue length of the model with finite buffer. A graphical presentation of the expected queue length and expected cost function are also carried out in section 5. Finally, concluding remarks are given in section 6.

2 Description of the Model

- Let $V_n(t-)$, $n = 0, 1, 2, ..., c-1$ denote the probability that the server is idle and there are n units waiting in the queue.

- $\Pi_{1,n}(t-)$, $n = a, a+1, a+2, ..., b-1$ denote the probability that the server is busy with an accessible batch and there are n units in the queue.

- $\Pi_{2,n}(t-)$, $n = 0, 1, 2, ...$ denote the probability that the server is busy with a non-accessible batch and there are n units in the queue (excluding those in service).

- Here the arriving customer may find the system in any of the following cases.

    1. (0, n), $(n = 0, 1, 2, ..., c-2)$ in this case the server is idle and the state of transition will be (0, n+1).
    2. (0, c-1), in this case the server is idle and the state of transition will be (1,c).
    3. (1, n), $(n = a, a+1, a+2, ..., b-2)$ in this case the server is busy with an accessible batch and the state of transition will be(1, n+1).
    4. (1, b-1), in this case the server is busy with an accessible batch and the state of transition will be(2, 0).
    5. (2, n), n=0,1,2,.... in this case the server is busy with a non-accessible batch and the state of transition will be (2, n+1).

3 Analysis of the Model with Infinite Buffer

Here the state space of the system is

$$\begin{aligned} S &= S_1 \cup S_2 \cup S_3, \\ \text{where} \quad S_1 &= \{(0,n), n = 0,1,...,c-1\}, \\ S_2 &= \{(1,n), n = a, a+1, a+2, ..., b-1\} \\ \text{and } S_3 &= \{(2,n), n = 0,1,2,...\} \end{aligned}$$

Hence the forward equations governing the transitions are

$$V_0(t+1) = (1-\lambda)V_0(t) + \mu(1-\lambda)\sum_{n=a}^{b-1}\Pi_{1,n}(t) + \mu(1-\lambda)\Pi_{2,0}(t) \tag{1}$$

$$V_n(t+1) = (1-\lambda)V_n(t) + \lambda V_{n-1}(t) + \mu(1-\lambda)\Pi_{2,n}(t), \quad n = 1,2,...,a-1 \tag{2}$$

$$V_n(t+1) = (1-\lambda)V_n(t) + \lambda V_{n-1}(t), \quad n = a, a+1, ..., c-1 \tag{3}$$

$$\Pi_{1,a}(t+1) = (1-\lambda)(1-\mu)\Pi_{1,a}(t) + \mu(1-\lambda)\Pi_{2,a}(t) \tag{4}$$

$$\Pi_{1,n}(t+1) = (1-\lambda)(1-\mu)\Pi_{1,n}(t) + \lambda(1-\mu)\Pi_{1,n-1}(t) + \mu(1-\lambda)\Pi_{2,n}(t), \quad n = a+1, a+2, ..., c-1 \tag{5}$$

$$\Pi_{1,c}(t+1) = (1-\lambda)(1-\mu)\Pi_{1,c}(t) + \lambda(1-\mu)\Pi_{1,c-1}(t) + \lambda(1-\mu)V_{c-1}(t) + \mu(1-\lambda)\Pi_{2,c}(t) \tag{6}$$

$$\Pi_{1,n}(t+1) = (1-\lambda)(1-\mu)\Pi_{1,n}(t) + \lambda(1-\mu)\Pi_{1,n-1}(t) + \mu(1-\lambda)\Pi_{2,n}(t), \quad n = c+1, c+2, ..., b-1 \tag{7}$$

$$\Pi_{2,0}(t+1) = (1-\lambda)(1-\mu)\Pi_{2,0}(t) + \mu(1-\lambda)\sum_{n=b}^{d}\Pi_{2,n}(t) + \lambda(1-\mu)\Pi_{1,b-1}(t) \tag{8}$$

$$\Pi_{2,n}(t+1) = (1-\lambda)(1-\mu)\Pi_{2,n}(t) + \lambda(1-\mu)\Pi_{2,n-1}(t) + \mu(1-\lambda)\Pi_{2,n+d}(t), \quad n = 1,2,3,... \tag{9}$$

3.1 Steady State Probabilities

The steady state probabilities of the system states are given by

$$\begin{aligned} V_n &= \lim_{t\to\infty} V_n(t), n = 0, 1, ..., c-1 \\ \Pi_{1,n} &= \lim_{t\to\infty} \Pi_{1,n}(t), n = a, a+1, a+2, ..., b-1 \text{ and} \\ \Pi_{2,n} &= \lim_{t\to\infty} \Pi_{2,n}(t), n = 0, 1, 2, ... \end{aligned}$$

From (3.1) to (3.9) the equations for the steady state probabilities are given by,

$$\lambda V_0 = \mu(1-\lambda)\sum_{n=a}^{b-1}\Pi_{1,n} + \mu(1-\lambda)\Pi_{2,0} \quad (10)$$

$$\lambda V_n = \lambda V_{n-1} + \mu(1-\lambda)\Pi_{2,n} \ , \quad n = 1, 2, ..., a-1 \quad (11)$$

$$\lambda V_n = \lambda V_{n-1}, \qquad n = a, a+1, ..., c-1 \quad (12)$$

$$(\lambda+\mu-\lambda\mu)\Pi_{1,a} = \mu(1-\lambda)\Pi_{2,a} \quad (13)$$

$$(\lambda+\mu-\lambda\mu)\Pi_{1,n} = \lambda(1-\mu)\Pi_{1,n-1} + \mu(1-\lambda)\Pi_{2,n}, \quad n = a+1, a+2, ..., c-1 \quad (14)$$

$$(\lambda+\mu-\lambda\mu)\Pi_{1,c} = \lambda(1-\mu)\Pi_{1,c-1} + \lambda(1-\mu)V_{c-1} + \mu(1-\lambda)\Pi_{2,c} \quad (15)$$

$$(\lambda+\mu-\lambda\mu)\Pi_{1,n} = \lambda(1-\mu)\Pi_{1,n-1} + \mu(1-\lambda)\Pi_{2,n}, \quad n = c+1, c+2, ..., b-1 \quad (16)$$

$$(\lambda+\mu-\lambda\mu)\Pi_{2,0} = \mu(1-\lambda)\sum_{n=b}^{d}\Pi_{2,n} + \lambda(1-\mu)\Pi_{1,b-1} \quad (17)$$

$$(\lambda+\mu-\lambda\mu)\Pi_{2,n} = \lambda(1-\mu)\Pi_{2,n-1} + \mu(1-\lambda)\Pi_{2,n+d}, \qquad n = 1, 2, 3, ... \quad (18)$$

Let $\Pi_{2,n} = z$, then the equation (3.18)can be written as

$$\mu(1-\lambda)z^{d+1} - (\lambda+\mu-\lambda\mu)z + \lambda(1-\mu) = 0 \quad (19)$$

Using Rouche's theorem there exist a unique positive real root less than unity of the equation (3.19) such that

$$\Pi_{2,n} = \Pi_{2,0}\, r^n, \quad n = 1, 2, 3, ... \quad (20)$$

From (3.14)

$$\begin{aligned} \Pi_{1,n} &= e_1\Pi_{1,n-1} + e_2\Pi_{2,n} \\ &= \Pi_{1,a}e_1^{n-a} + \Pi_{2,0}e_2 r^{a+1}\left[\frac{e_1^{n-a} - r^{n-a}}{e_1 - r}\right]; \quad n = a+1, a+2, ..., c-1 \\ &\text{where, } e_1 = \frac{\lambda(1-\mu)}{\lambda+\mu-\lambda\mu}; \ \ e_2 = \frac{\mu(1-\lambda)}{\lambda+\mu-\lambda\mu} \end{aligned}$$

From (3.13)

$$\Pi_{1,a} = \Pi_{2,0} e_2 . r^a \tag{21}$$

$$\therefore \Pi_{1,n} = \Pi_{2,0} e_2 r^a \left[e_1^{n-a} + \frac{r}{e_1 - r} \left(e_1^{n-a} - r^{n-a} \right) \right], \quad n = a+1, a+2, ..., c-1 \tag{22}$$

From (3.16)

$$\begin{aligned} \Pi_{1,n} &= e_1 \Pi_{1,n-1} + e_2 \Pi_{2,n} \\ &= \Pi_{1,c} e_1^{n-c} + \Pi_{2,0} e_2 r^{c+1} \left[\frac{e_1^{n-c} - r^{n-c}}{e_1 - r} \right], \quad n = c+1, c+2, ..., b-1 \end{aligned} \tag{23}$$

Using (3.16) and (3.19) ,

$$\Pi_{1,c} = \Pi_{2,0} D_1 \tag{24}$$

where, $D_1 = \dfrac{1}{e_1^{b-c}} \left\{ 1 - e_2 \left[\dfrac{r^b - r^d}{1 - r} + e_1 r^{c+1} \left(\dfrac{e_1^{b-1-c} - r^{b-1-c}}{e_1 - r} \right) \right] \right\}$

$$\therefore \Pi_{1,n} = \Pi_{2,0} \left[D_1 e_1^{n-c} + e_2 r^{c+1} \left(\frac{e_1^{n-c} - r^{n-c}}{e_1 - r} \right) \right], \quad n = c+1, c+2, ..., b-1 \tag{25}$$

From (3.10) to (3.12) we get,

$$V_0 = \Pi_{2,0} D_2 \tag{26}$$

$$\begin{aligned} \text{where, } D_2 = 1 + e_3 \Bigg\{ & \left(\frac{1 - e_1^{c-a}}{1 - e_1} \right) (e_2 r^a + D_1) \\ & + \left(\frac{e_2}{e_1 - r} \right) \left[r^{a+1} \left(\frac{e_1 (1 - e_1^{c-1-a})}{1 - e_1} - \frac{r(1 - r^{c-1-a})}{1 - r} \right) \right. \\ & \left. + r^{c+1} \left(\frac{e_1 (1 - e_1^{b-1-c})}{1 - e_1} - \frac{r(1 - r^{b-1-c})}{1 - r} \right) \right] \Bigg\} \end{aligned}$$

$$V_n = \Pi_{2,0} \left[D_2 + e_3 \frac{r(1 - r^n)}{1 - r} \right], \quad n = 1, 2, 3, ..., a-1 \tag{27}$$

$$V_n = \Pi_{2,0} \left[D_2 + e_3 \frac{r(1 - r^{a-1})}{1 - r} \right], \quad n = a, a+1, ..., c-1 \tag{28}$$

Then $\Pi_{2,0}$ can be obtained by using the normalizing condition,

$$\sum_{n=0}^{c-1} V_n + \sum_{n=a}^{b-1} \Pi_{1,n} + \sum_{n=0}^{\infty} \Pi_{2,n} = 1$$

$$\begin{aligned}
\therefore \Pi_{2,0} &= \left\{ 1 + D_1 \left(1 + \frac{e_1 - e_1^{b-c}}{1 - e_1} \right) + cD_2 + r(1-r)^{-1} \right. \\
&\quad + e_2 \left\{ r^a \left[1 + \left(\frac{e_1 - e_1^{c-a}}{1 - e_1} \right) \left(1 + \frac{r}{e_1 - r} \right) - \left(\frac{r}{e_1 - r} \right) \left(\frac{r - r^{c-a}}{1 - r} \right) \right] \right. \\
&\quad \left. + \frac{r^{c+1}}{e_1 - r} \left[\left(\frac{e_1 - e_1^{b-c}}{1 - e_1} \right) - \left(\frac{r - r^{b-c}}{1 - r} \right) \right] \right\} \\
&\quad \left. + e_3 \frac{r}{1 - r} \left[(a-1) + (1 - r^{a-1}) \left((c-a) - \frac{r}{1 - r} \right) \right] \right\}^{-1} \qquad (29)
\end{aligned}$$

3.2 Expected Queue Length

In this model the expected queue length L_q is given by

$$\begin{aligned}
L_q &= \sum_{n=1}^{c-1} n.V_n + \sum_{n=1}^{\infty} n.\Pi_{2,n} \\
&= \Pi_{2,0} \left\{ D_2 \frac{c(c-1)}{2} + r(1-r)^{-2} - r^{a-1} \left(\frac{c(c-1) - a(a-1)}{2} \right) \right. \\
&\quad \left. + e_3 \frac{r}{1 - r} \left[\frac{c(c-1)}{2} + ar^{a-1}(1-r)^{-1} - (r - r^a)(1-r)^{-2} \right] \right\} \qquad (30)
\end{aligned}$$

3.3 Determination of the Optimal Control Limits

The optimal control limits can be obtained by minimizing expected cost function. Here we assume that costs are charged for serving the customers and for holding them in the queue. Let C_0 be the over head cost to initiate a batch service, C_h be the holding cost per customer in the queue in steady state, C_1 be the cost per batch in the case of accessible batch service and C_2 be the service cost per batch in the case of non-accessible batch, independent of batch size.

The expected cost function is given by

$$\begin{aligned}
E(cost) &= C_0 + C_h L_q + C_1 \sum_{n=a}^{b-1} \Pi_{1,n} \\
&\quad + \sum_{k=0}^{\infty} \left\{ \sum_{n=0}^{b-1} (kC_2 + C_1) + \sum_{n=b}^{d} (k+1)C_2 \right\} \Pi_{2,kd+n} \\
&= C_0 + C_h L_q + C_1 \Pi_{2,0} \left\{ D_1 \left(1 + \frac{e_1 - e_1^{b-c}}{1 - e_1} \right) \right. \\
&\quad + e_2 \left[r^a \left\{ 1 + \left(\frac{e_1 - e_1^{c-a}}{1 - e_1} \right) \left(1 + \frac{r}{e_1 - r} \right) - \left(\frac{r}{e_1 - r} \right) \left(\frac{r - r^{c-a}}{1 - r} \right) \right\} \right. \\
&\quad \left. \left. + \frac{r^{c+1}}{e_1 - r} \left(\frac{e_1 - e_1^{b-c}}{1 - e_1} - \frac{r - r^{b-c}}{1 - r} \right) \right] \right\} \\
&\quad + \sum_{k=0}^{\infty} \left[\sum_{n=0}^{b-1} (kC_2 + C_1) + \sum_{n=b}^{d} (k+1)C_2 \right] \Pi_{2,kd+n} \qquad (31)
\end{aligned}$$

3.4 Particular Cases

1. When a=c and interchanging b and d we get (a, b) policy bulk service queue with an accessible limit d.

2. When a=c=b and interchanging b and d we get (a, b) policy bulk service queue.

4 The Model with Finite Buffer

Here we assume that the waiting space is limited and the maximum number of customers allowed in the queue at any time is N. It is assumed that the waiting capacity N is greater than twice the service capacity. That is N is greater than $2b$.

The difference equation for the system states are,

$$V_0(t+1) = (1-\lambda)V_0(t) + \mu(1-\lambda)\sum_{n=a}^{b-1}\Pi_{1,n}(t) + \mu(1-\lambda)\Pi_{2,0}(t) \quad (32)$$

$$V_n(t+1) = (1-\lambda)V_n(t) + \lambda V_{n-1}(t) + \mu(1-\lambda)\Pi_{2,n}(t), \quad n = 1,2,...,a-1 \quad (33)$$

$$V_n(t+1) = (1-\lambda)V_n(t) + \lambda V_{n-1}(t), \quad n = a, a+1, ..., c-1 \quad (34)$$

$$\Pi_{1,a}(t+1) = (1-\lambda)(1-\mu)\Pi_{1,a}(t) + \mu(1-\lambda)\Pi_{2,a}(t) \quad (35)$$

$$\Pi_{1,n}(t+1) = (1-\lambda)(1-\mu)\Pi_{1,n}(t) + \lambda(1-\mu)\Pi_{1,n-1}(t) + \mu(1-\lambda)\Pi_{2,n}(t), \quad n = a+1, a+2, ..., c-1 \quad (36)$$

$$\Pi_{1,c}(t+1) = (1-\lambda)(1-\mu)\Pi_{1,c}(t) + \lambda(1-\mu)\Pi_{1,c-1}(t) + \lambda(1-\mu)V_{c-1}(t) + \mu(1-\lambda)\Pi_{2,c}(t) \quad (37)$$

$$\Pi_{1,n}(t+1) = (1-\lambda)(1-\mu)\Pi_{1,n}(t) + \lambda(1-\mu)\Pi_{1,n-1}(t) + \mu(1-\lambda)\Pi_{2,n}(t), \quad n = c+1, c+2, ..., b-1 \quad (38)$$

$$\Pi_{2,0}(t+1) = (1-\lambda)(1-\mu)\Pi_{2,0}(t) + \mu(1-\lambda)\sum_{n=b}^{d}\Pi_{2,n}(t) + \lambda(1-\mu)\Pi_{1,b-1}(t) \quad (39)$$

$$\Pi_{2,n}(t+1) = (1-\lambda)(1-\mu)\Pi_{2,n}(t) + \lambda(1-\mu)\Pi_{2,n-1}(t) + \mu(1-\lambda)\Pi_{2,n+d}(t), \quad n = 1,2,3,...,N-d \quad (40)$$

$$\Pi_{2,n}(t+1) = (1-\lambda)(1-\mu)\Pi_{2,n}(t) + \lambda(1-\mu)\Pi_{2,n-1}(t), \quad n = N-d+1, N-d+2, ..., N \quad (41)$$

4.1 Steady State Probabilities

The steady state probabilities of the system states are given by

$$V_n = \lim_{t\to\infty} V_n(t), n = 0,1,...,c-1$$
$$\Pi_{1,n} = \lim_{t\to\infty} \Pi_{1,n}(t), n = a, a+1, a+2, ..., b-1 \text{ and}$$
$$\Pi_{2,n} = \lim_{t\to\infty} \Pi_{2,n}(t), n = 0,1,2,...$$

From (4.1) to (4.10) the equations for the steady state probabilities are given by,

$$\lambda V_0 = \mu(1-\lambda)\sum_{n=a}^{b-1}\Pi_{1,n} + \mu(1-\lambda)\Pi_{2,0} \tag{42}$$

$$\lambda V_n = \lambda V_{n-1} + \mu(1-\lambda)\Pi_{2,n} \quad , \quad n = 1, 2, ..., a-1 \tag{43}$$

$$\lambda V_n = \lambda V_{n-1}, \qquad n = a, a+1, ..., c-1 \tag{44}$$

$$(\lambda+\mu-\lambda\mu)\Pi_{1,a} = \mu(1-\lambda)\Pi_{2,a} \tag{45}$$

$$(\lambda+\mu-\lambda\mu)\Pi_{1,n} = \lambda(1-\mu)\Pi_{1,n-1} + \mu(1-\lambda)\Pi_{2,n}, \quad n = a+1, a+2, ..., c-1 \tag{46}$$

$$(\lambda+\mu-\lambda\mu)\Pi_{1,c} = \lambda(1-\mu)\Pi_{1,c-1} + \lambda(1-\mu)V_{c-1} + \mu(1-\lambda)\Pi_{2,c} \tag{47}$$

$$(\lambda+\mu-\lambda\mu)\Pi_{1,n} = \lambda(1-\mu)\Pi_{1,n-1} + \mu(1-\lambda)\Pi_{2,n}, \quad n = c+1, c+2, ..., b-1 \tag{48}$$

$$(\lambda+\mu-\lambda\mu)\Pi_{2,0} = \mu(1-\lambda)\sum_{n=b}^{d}\Pi_{2,n} + \lambda(1-\mu)\Pi_{1,b-1} \tag{49}$$

$$(\lambda+\mu-\lambda\mu)\Pi_{2,n} = \lambda(1-\mu)\Pi_{2,n-1} + \mu(1-\lambda)\Pi_{2,n+d}, \quad n = 1, 2, 3, ..., N-d \tag{50}$$

$$(\lambda+\mu-\lambda\mu)\Pi_{2,n} = \lambda(1-\mu)\Pi_{2,n-1}, \quad n = N-d+1, N-d+2, ..., N \tag{51}$$

From (4.11) to (4.20) it is clear that the finite buffer situation affects only the value of $\Pi_{2,n}, \quad n = 1, 2, 3, ...N$

Let $\Pi_{2,n} = Z$ then the equation (4.19) can be written as a difference equation,

$$h(Z) = \mu(1-\lambda)Z^{d+1} - (\lambda+\mu-\lambda\mu)Z + \lambda(1-\mu) = 0$$

Then using Rouche's theorem and solving h(Z)=0, we get,

$$\Pi_{2,n} = \Pi_{2,0}r^n, \qquad n = 1, 2, 3, ..., N-d \tag{52}$$

where r is the unique positive real root less than unity of the equation, $h(Z) = 0$

$$\Pi_{2,n} = \Pi_{2,0}e_1^{n-N+d}r^{N-d}, n = N-d+1, N-d+2, ..., N \tag{53}$$

where, $\quad e_1 = \dfrac{\lambda(1-\mu)}{\lambda+\mu-\lambda\mu}$ and

$$V_0 = \Pi_{2,0} D_2 \tag{54}$$

$$V_n = \Pi_{2,0}\left[D_2 + e_3\frac{r(1-r^n)}{1-r}\right], \quad n = 1,2,3,...,a-1 \tag{55}$$

$$V_n = \Pi_{2,0}\left[D_2 + e_3\frac{r(1-r^{a-1})}{1-r}\right], \quad n = a, a+1, ..., c-1 \tag{56}$$

$$\Pi_{1,a} = \Pi_{2,0} e_2 r^a \tag{57}$$

$$\Pi_{1,n} = \Pi_{2,0} e_2 r^a \left[e_1^{n-a} + \frac{r}{e_1 - r}\left(e_1^{n-a} - r^{n-a}\right)\right], n = a+1, a+2, ..., c-1 \tag{58}$$

$$\Pi_{1,c} = \Pi_{2,0} D_1 \tag{59}$$

$$\Pi_{1,n} = \Pi_{2,0}\left[D_1 e_1^{n-c} + e_2 r^{c+1}\left(\frac{e_1^{n-c} - r^{n-c}}{e_1 - r}\right)\right], n = c+1, c+2, ..., b-1 \tag{60}$$

Then using the normalizing condition $\sum_{n=0}^{c-1} V_n + \sum_{n=a}^{b-1} \Pi_{1,n} + \sum_{n=0}^{N} \Pi_{2,n} = 1$, we get,

$$\begin{aligned}
\Pi_{2,0} = \Bigg\{ & 1 + D_1\left(1 + \frac{e_1 - e_1^{b-c}}{1-e_1}\right) + cD_2 \\
& + e_2\left\{r^a\left[1 + \left(\frac{e_1 - e_1^{c-a}}{1-e_1}\right)\left(1 + \frac{r}{e_1 - r}\right) - \left(\frac{r}{e_1 - r}\right)\left(\frac{r - r^{c-a}}{1-r}\right)\right]\right. \\
& \left. + \frac{r^{c+1}}{e_1 - r}\left[\left(\frac{e_1 - e_1^{b-c}}{1-e_1}\right) - \left(\frac{r - r^{b-c}}{1-r}\right)\right]\right\} \\
& + e_3\frac{r}{1-r}\left[(c-a)(1-r^{a-1}) + (a-1) - \left(\frac{r - r^a}{1-r}\right)\right]\Bigg\}^{-1}
\end{aligned} \tag{61}$$

4.2 Expected Queue Length

In this model the expected queue length L_q is given by

$$\begin{aligned}
L_q = \Pi_{2,0}\Bigg\{ & D_2\frac{c(c-1)}{2} + e_3\frac{r}{1-r}\left[\frac{c(c-1)}{2} - r(1-r)^{-2}(1 + r^{a-1})\right. \\
& \left. + r^{a-1}\left(a(1-r)^{-1} - \frac{c(c-1) - a(a-1)}{2}\right)\right] \\
& + r(1-r)^{-2}[1 - r^{N-d}((N-d)(1-r) + 1)] \\
& + r^{N-d} e_1 (1-e_1)^{-1}\left[(N-d)(1 - e_1^{d-1}) - de_1^{d-1} + (1 - e_1^d)(1-e_1)^{-1}\right]\Bigg\}
\end{aligned} \tag{62}$$

5 Numerical Illustration

The expected queue length computed using the equation (3.30) is plotted for different values of c, b, λ, μ and a when d=50. The graphs obtained are given below.

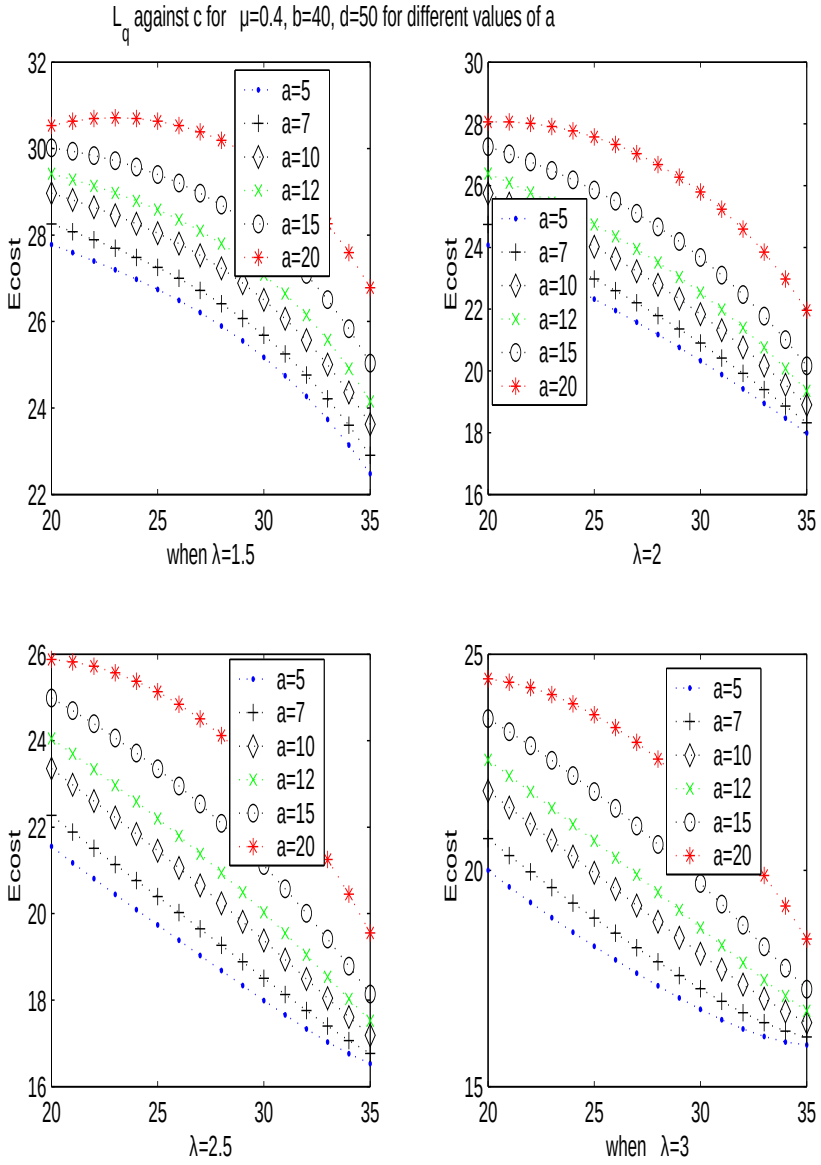


Figure 5.1

Remark: Here it can be noted that the control limits a and c affects the expected queue length.

The expected cost function computed using the equation (3.31) is plotted for different values of a, c, b λ, μ, when d=50. The graphs obtained are given below.

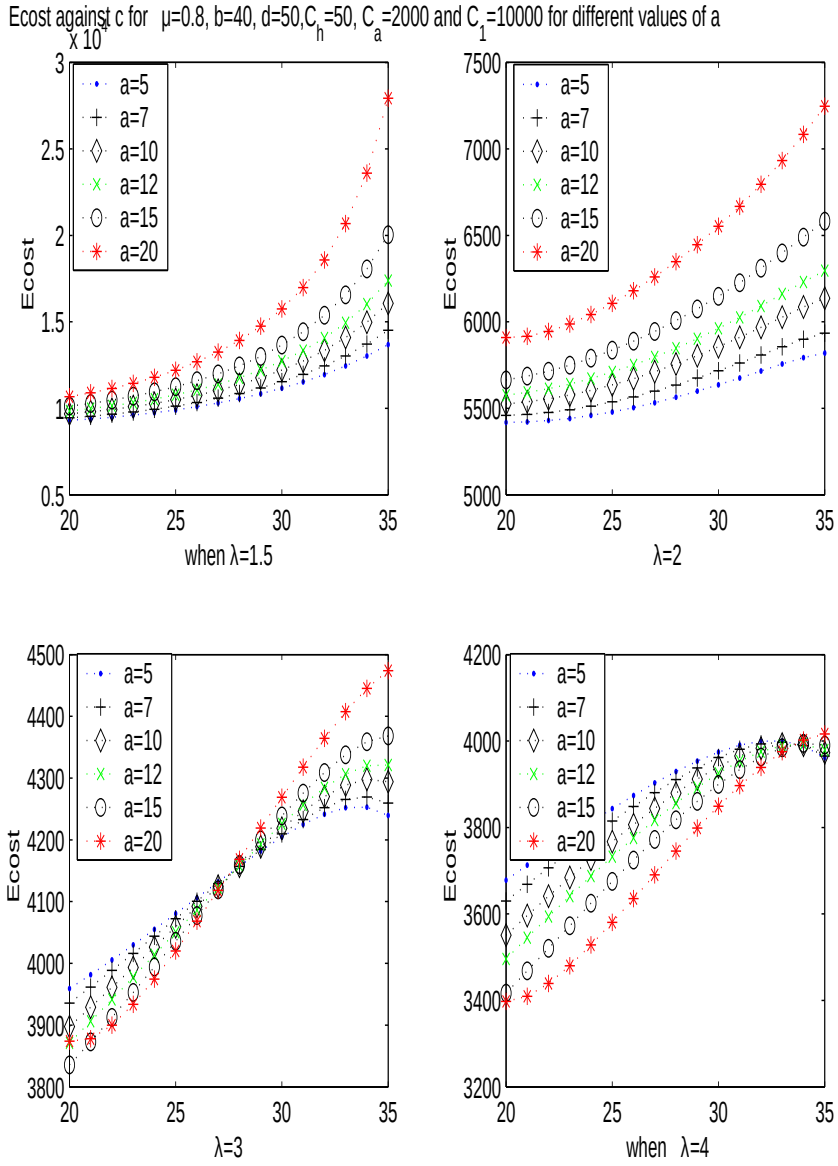


Figure 5.2

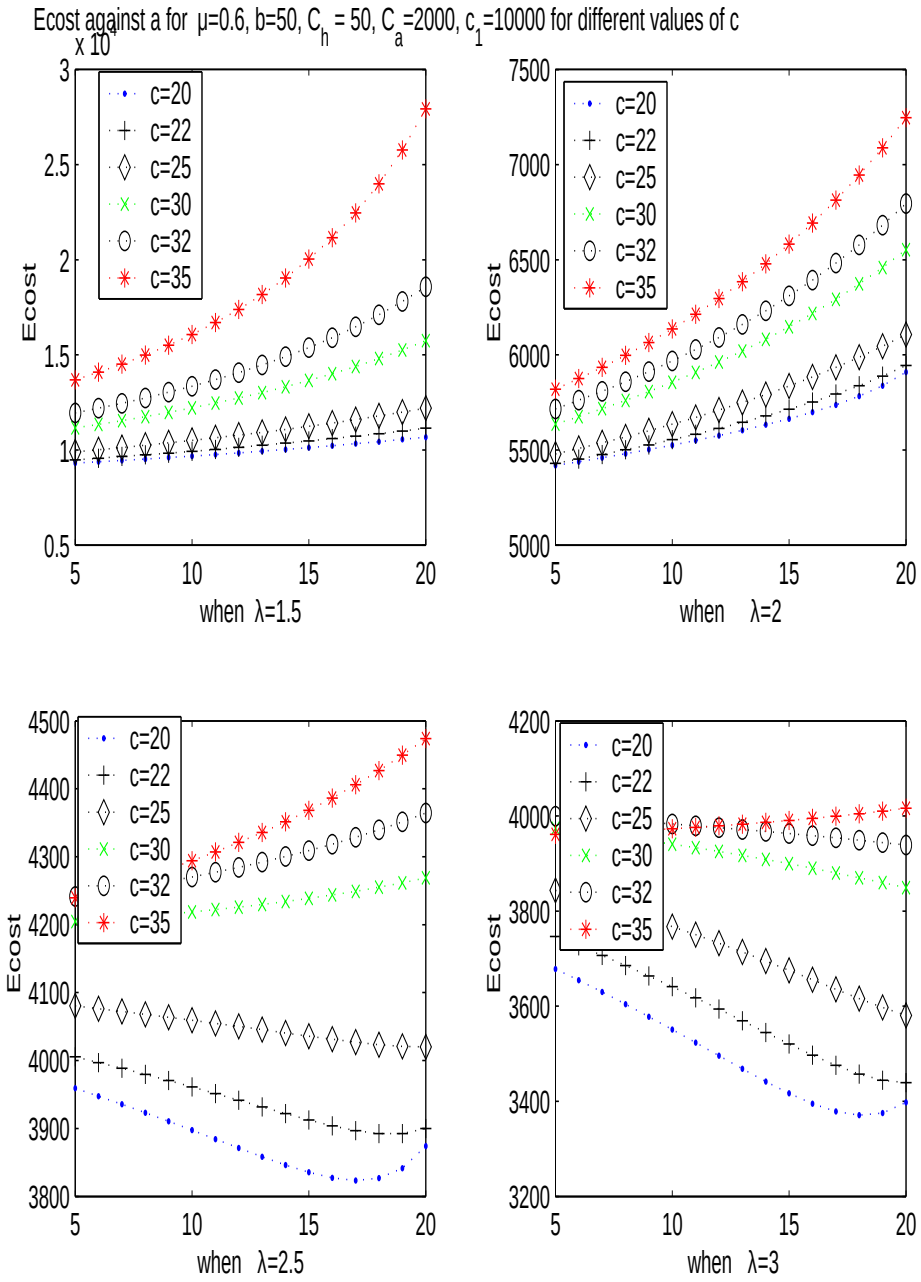


Figure 5.3

Remark: It can be noted that optimal control limits a, b and c can be determined by minimising the expected cost function. In the Figure (5.3), when λ=3, the

optimal values of a, and c are 18 and 20 respectively. That is when $a < c$.

6 Conclusion

An (a, c, d) policy discrete time bulk service queue with accessible and non-accessible batches has been studied in this article. Analysis of the model is done and expressions for steady state probabilities and expected queue length, for finite and infinite buffer models are obtained. It can be noted that by minimising the expected cost function, optimal control limits a, b and c can be obtained.

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